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Formalization of Petri Nets with Individual Tokens as Basis for DPO Net Transformations

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Abstract: Reconfigurable place/transition systems are Petri nets with initial markings and a set of rules which allow the modification of the net structure during runtime. They have been successfully used in different areas like mobile ad-hoc networks. In most of these applications the modification of net markings during runtime is an important issue. This requires the analysis of the interaction between firing and rule-based modification. For place/transition systems this analysis has been started explicitly without using the general theory of $\mathcal{M}$-adhesive transformation systems, because firing cannot be expressed by rule-based transformations for P/T systems in this framework. This problem is solved in this paper using the new approach of P/T nets with individual tokens. In our main results we show that on one hand this new approach allows to express firing by transformation via suitable transition rules. On the other hand transformations of P/T nets with individual tokens can be shown to be an instance of $\mathcal{M}$-adhesive transformation systems, such that several well-known results, like the local Church-Rosser theorem, can be applied. This avoids a separate conflict analysis of token firing and transformations. Moreover, we compare the behavior of P/T nets with individual tokens with that of classical P/T nets. Our new approach is also motivated and demonstrated by a network scenario modeling a distributed communication system.

Keywords: Petri net transformation, reconfigurable place/transition systems, Petri nets with individual tokens, collective token approach, network scenario

1 Introduction

Petri nets are one of the main formalisms to describe and analyze concurrent processes. They have been a promising candidate for formal extensions on the one hand, but on the other hand

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also for integrations with different formal techniques to capture more complex aspects.

A theory of rule-based transformation based on double pushout (DPO) graph transformation [EEPT06] is available for place/transition (P/T) systems, i. e. P/T nets with an initial marking. This transformation of P/T systems changing their net structure has been successfully used for modeling adaptive workflows and mobile ad-hoc networks in [HME05, BDHM06].

P/T systems have been shown to form a weak adhesive high-level replacement (HLR) category with the class of all marking-strict morphisms [PEHP08]. This allows us to apply all the results for weak adhesive HLR transformation systems concerning the local Church-Rosser theorem, parallelism, and concurrency of transformations as shown in [EEPT06] also to transformation systems of P/T systems. In this paper, we use the notion of $\mathcal{M}$-adhesive category [EGH10] which is short for vertical weak adhesive HLR category. In $\mathcal{M}$-adhesive categories Van Kampen (VK) squares only need to satisfy the vertical VK-property, i. e. the VK-property has to hold for cubes where the vertical morphisms are in $\mathcal{M}$. In contrast, for a weak adhesive HLR categories it is required that the VK-property does also hold for cubes, where the horizontal morphisms are in $\mathcal{M}$. However, as shown in [EGH10] all the main results of [EEPT06] are still valid for $\mathcal{M}$-adhesive categories.

The concept of Petri systems leads to a category **PTSys** with morphisms allowing to increase the number of tokens on corresponding places. Unfortunately, $(\mathbf{PTSys}, \mathcal{M}_{inj})$ with the class $\mathcal{M}_{inj}$ of all injective morphisms is not $\mathcal{M}$-adhesive in contrast to $(\mathbf{PTSys}, \mathcal{M}_{strict})$, where $\mathcal{M}_{strict}$ is the class of injective morphisms where the number of tokens on corresponding places is equal [Pra08]. Using marking-strict morphisms, we can not formulate adequate transformation rules for P/T systems that change markings. This is inconvenient because marking-changing rules are essential to express token firing by transformation via suitable transition rules and for modeling communication systems and platforms with Petri nets, especially for realizing multicasting of data tokens in high-level nets [BEE+09].

To overcome this restriction, we present a new Petri net formalism, called “place/transition nets with individual tokens” or short PTI nets, together with a rule-based transformation approach. The difference between PTI nets and P/T systems concerns the representation of net markings: for the new individual approach, we propose a net’s marking as a set of individuals instead of a (collective) sum of a monoid. The formal definition of nets with individual tokens still follows the concept “Petri nets are monoids” from [MM90].

The paper is structured as follows: Section 2 introduces PTI nets, their firing behavior, and rule-based transformation of PTI nets based on graph transformation with double pushouts. We demonstrate that the new concept of P/T nets with individual tokens is compatible with the concept of P/T systems using a construction that maps PTI nets to corresponding P/T systems. For this purpose we define an equivalence relation on the class of PTI nets, such that the equivalence classes are in one-to-one correspondence to the P/T systems. Moreover, we show that the construction preserves and reflects the firing behavior. As a running example, we demonstrate a simple network model that can be reconfigured by rule applications that add new clients to the network.

As first main result we show in Section 3 that the category of PTI nets with the class of all injective morphisms forms an $\mathcal{M}$-adhesive category which allows to formulate marking changing rules. This important result is the basis for further results concerning analysis. First, we formulate a necessary and sufficient gluing condition for the applicability of transformation rules in the given $\mathcal{M}$-adhesive category of PTI nets. Then, we demonstrate the equivalence of firing steps with corresponding transition firing rules. The second main result shows that token firing can be expressed by rule-based transformation based on suitable transition rules, leading to a local Church-Rosser theorem for rule applications and firing steps.

In the concluding Section 4, we give an outlook on algebraic high-level nets with individual tokens for modeling especially highly dynamic structures and complex behavior in the area of communication platforms in an adequate way.

2 P/T Nets with Individual Tokens

In this section we introduce our new concept of P/T nets with individual tokens (PTI nets) and compare it to the classical concept of P/T systems with initial markings. Furthermore, we define a rule-based transformation of PTI nets in the sense of rule-based graph transformation [EEPT06]. As an example, we demonstrate a simple model of a distributed reconfigurable network.

2.1 P/T Nets with Individual Tokens and their Relationship to P/T Systems

The notion of nets with individual tokens was mentioned first in [Rei85] where it was used to describe "tokens that can be identified as individual objects". The main contribution of that article was the definition of markings as multisets of distinguished elements rather than amounts of indistinguishable black tokens. In the end, individual tokens in that context is a synonym for what by now is known as data tokens in high-level nets.

Further, there is the notion of token individuality that has been coined in [GP95] as "individual token interpretation" of firing steps, which entitles the definition of processes from [GR83]. Under the individual approach, firing sequences consider not only the number and value of tokens (as in the collective approach) but also their history of tokens. In [vG05], the author investigates the collective/individual dichotomy of firing steps and the expressive power of the different firing rules w. r. t. labeled transition step systems. [BMMS99] formalizes the individual token interpretation of firing steps categorically with a functorial individual firing semantics.

We try to combine aspects of both approaches dealing with individual tokens. On the one hand, we need a concept of individual tokens on the syntactical level of Petri systems like in [Rei85] in order to gain benefits for the transformation of marked Petri nets. With such individual tokens, rules can match specific tokens which allows us to formulate rules for manipulating markings freely without necessarily changing the net's structure as in the category **PTSys** of P/T systems, i. e. P/T nets with collective markings (cf. [EHP+07]). On the other hand, we need individual "black" tokens like in [GP95] without presuming different data values for the tokens, because we also want to have low-level Petri nets with individual tokens.

For this purpose, we introduce the new notion of place/transition nets with individual tokens (PTI nets), their firing behavior, and application of PTI transformation rules.

Definition 1 (Place/Transition Nets with Individual Tokens (PTI)) We define a marked P/T net with individual tokens, short PTI net, as $NI = (PN, I, m)$, where

- $PN = (P, T, \ pre, post : \ T \to P^{\oplus})$ is a classical P/T net, where $P^{\oplus}$ is the free commutative monoid over P,

- I is the finite set of individual tokens of NI, and
- $m\colon I \to P$ is the marking function, assigning the individual tokens to the places.

Further, we denote the environment of a transition $t \in T$ as

$$ENV(t) = \{p \in P \mid pre(t)(p) \neq 0 \vee post(t)(p) \neq 0\} \subseteq P$$

Example 1 (Place/Transition Nets with Individual Tokens) *Figure 1 shows an example of a PTI net modeling a simple network which consists of several clients. These clients can communicate with each other only indirectly via switches by sending or receiving data packages represented by black tokens. If a client wants to send data to another client which is connected to a different switch then it first has to send the data to the switch to which it is connected. The switch can then forward the data to the respective other switch which sends the data to the addressee. Each client Clientx has a complement place Cx which represents the free data capacity of this client. The net has a marking of individual tokens* $I = \{i1,\ldots,i7\}$*. The individual tokens are mapped to the corresponding places by a marking function* m *with* $m(i1) = C2$, $m(i2) = m(i3) = m(i4) = Client1$, $m(i5) = m(i6) = Client2$ *and* $m(i7) = Switch1$*.*

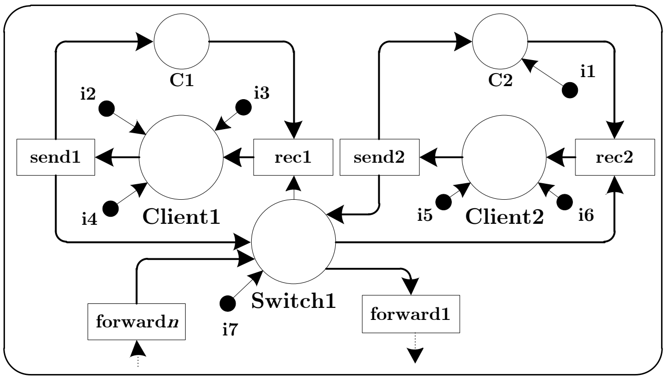


Figure 1: PTI net $(SimpleNetwork, I, m)$

Every P/T net with individual tokens corresponds to a P/T system in the collective approach as defined in [EHP+07]. The following construction *Coll* flattens a PTI net to a P/T system with collective marking by forgetting the individuality of token elements.

Definition 2 (Collective Construction for PTI Nets) Given a PTI net $NI = (PN, I, m)$, we define $Coll(NI) = (PN, \mu)$ where $\mu = \sum_{i\in I} m(i) \in P_{PN}^{\oplus}$.

Note that we can denote the collective marking alternatively as the sum with explicit coefficients $\mu = \sum_{p\in P_{PN}} |m^{-1}(p)| \cdot p$.

Next, we define an equivalence relation $\approx$ on PTI nets and show that two PTI nets are equivalent if and only if they correspond to the same P/T system with collective marking. Moreover, we show that for every collective P/T system there is at least one corresponding PTI net. This allows us to show that our individual approach and the collective approach are compatible with each other, in the sense that the class *PTSys* of all P/T systems corresponds bijectively to the quotient $PTINets/_{\approx}$ where all equivalent PTI nets are identified.

Definition 3 (Equivalence of PTI Nets) We call two PTI nets $NI = (PN, I, m)$ and $NI' = (PN', I', m')$ equivalent and write $NI \approx NI'$, if $PN = PN'$ and there exists a bijective function $f : I \to I'$ with $m' \circ f = m$.

Note that because bijective functions are closed under composition and inversion, $\approx$ is an equivalence relation.

Lemma 1 (Collective Equality and Equivalence) *For any two PTI nets* $NI = (PN, I, m)$ *and* $NI' = (PN', I', m')$ *hold the equivalence*

$$Coll(NI) = Coll(NI') \Leftrightarrow NI \approx NI'$$

Proof. We assume $Coll(NI) = (PN, \mu)$, $Coll(NI') = (PN', \mu')$, and that P is the set of places of PN (and also of PN').

"$\Rightarrow$": From $Coll(NI) = Coll(NI')$ we get $PN = PN'$ and $\sum_{i \in I} m(i) = \mu = \mu' = \sum_{i \in I'} m'(i)$. We construct a bijection $f : I \to I'$ compatible with m and m'.

Choose for each place $p \in P$ an arbitrary bijection $f_p : m^{-1}(p) \to m'^{-1}(p)$ between the subsets of tokens of I and I' that are mapped to p by m and m', respectively. Such bijections exist because from the $\mu = \mu'$ we get by the equality of their coefficients for all $p \in P$ that $|m^{-1}(p)| = |m'^{-1}(p)|$. Consider the function $f : I \to I'$ with $f(x) = f_p(x)$ for $x \in m^{-1}(p)$, which is well-defined because $I = \bigcup_{p \in P} m^{-1}(p)$, $I' = \bigcup_{p \in P} m'^{-1}(p)$ and the preimage subsets of m and m' are disjoint. Moreover, f is bijective and for all $p \in P$ and all $x \in m^{-1}(p)$ we have $m' \circ f(x) = m'(f_p(x)) = p = m(x)$ from which we conclude $NI \approx NI'$.

"$\Leftarrow$": From $NI \approx NI'$, we get $PN = PN'$ and bijective $f : I \to I'$ with $m' \circ f = m$. We have to show that $\mu = \mu'$:

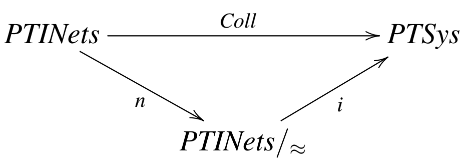
$$\mu = \sum_{i \in I} m(i) = \sum_{i \in I} m' \circ f(i) = \sum_{p \in P} |(m' \circ f)^{-1}(p)| \cdot p \overset{f \text{bij.}}{=} \sum_{p \in P} |m'^{-1}(p)| \cdot p = \sum_{i \in I'} m'(i) = \mu'.$$

□

Lemma 2 (*Coll* is surjective) *For every P/T system* (PN, μ)*, there is a PTI net NI with* $Coll(NI) = (PN, \mu)$.

Proof. Let P be the set of places of PN. For $\mu = \sum_{p \in P} \lambda_p \cdot p$, consider for each $p \in P$ a set I_p of λ_p elements with all I_p being mutually disjoint. We choose $NI = (PN, I, m)$ with $I = \bigcup_{p \in P} I_p$ and $m : I \to P$ with $m(x) = p$ for $x \in I_p$. Hence, $Coll(NI) = (PN, \hat{\mu})$ with $\hat{\mu} = \sum_{i \in I} m(i) = \sum_{p \in P} \sum_{i \in I_p} m(i) = \sum_{p \in P} \lambda_p \cdot p = \mu$. □

Theorem 1 *The quotient* $PTINets/_{\approx}$ *of the class of all PTI nets by their equivalence relation corresponds bijectively to the class PTSys of all P/T systems.*



Proof. Consider the function $i\colon PTINets/_{\approx} \to PTSys$ with $i([NI]_{\approx}) = Coll(NI)$. Note that $i \circ n = Coll$, where n is the natural function mapping a PTI net to its equivalence class. By Lemma 1, we get that i is well-defined and injective because all PTI nets in the same equivalence class have the same collective construction to which only elements of this particular equivalence class are mapped by $Coll$ and i, respectively. By Lemma 2, i is also surjective and hence bijective. □

2.2 Firing of P/T Nets with Individual Tokens

Now, we define firing steps of transitions in PTI nets. Due to the fact that the tokens have identities, we have to consider a possible firing step in the context of a specific selection of tokens because there may be several valid firing steps for a transition under a particular marking.

Definition 4 (Firing of PTI Nets) A transition $t \in T$ in a PTI net NI = (P,T,pre,post,I,m) is *enabled* under a *token selection* (M,m,N,n), where

- $M \subseteq I$, m is the token mapping of NI,
- N is a set with $(I \setminus M) \cap N = \emptyset$, $n\colon N \to P$ is a function,

if it meets the *token selection condition* $\sum_{i \in M} m(i) = pre(t) \wedge \sum_{i \in N} n(i) = post(t)$

If an enabled transition t fires, the follower marking (I', m') is given by

$$I' = (I \setminus M) \cup N, \quad m' : I' \to P \text{ with } m'(x) = \begin{cases} m(x), & x \in I \setminus M \\ n(x), & x \in N \end{cases}$$

leading to $NI' = (P, T, pre, post, I', m')$ as the new PTI net in the *firing step* $NI \xrightarrow{t,S} NI'$ via $S = (M, m, N, n)$.

Remark 1 (Token Selection) *The purpose of the token selection is to specify exactly which tokens should be consumed and produced in the firing step. Thus, $M \subseteq I$ selects the individual tokens to be consumed, and N contains the set of individual tokens to be produced. Clearly, $(I \setminus M) \cap N = \emptyset$ must hold because it is impossible to add an individual token to a net that already contains this token. m and n relate the tokens to their carrying places. It would be sufficient to consider just the restriction $m_{|M}$ here or to infer it from the net but as a compromise on symmetry and readability we denote m in the token selection.*

Example 2 (Firing of PTI Nets) *Consider again the PTI net $(SimpleNetwork, I, m)$ in Figure 1. We want to fire the transition send2 to send one data package from Client2 to the switch. Even though we have only black tokens, we have to choose which of the tokens on the place Client2 should be consumed by the transition, because the tokens have identities. We decide to take* *the token i6. So we have a token selection* $S = (M,m,N,n)$ *with* $M = \{i6\}$, $m(i6) = Client2$, $N = \{i8,i9\}$, $n(i8) = C2$ *and* $n(i9) = Switch1$.

Now, $send2$ *is enabled under selection* S *because there is*

$$m(i6) = Client2 = pre(send2) \quad \text{and} \quad n(i8) \oplus n(i9) = C2 \oplus Switch1 = post(send2)$$

which means that it meets the token selection condition. Hence, there is a firing step

$$(SimpleNetwork, I, m) \xrightarrow{send2,S} (SimpleNetwork, I', m')$$

with $I' = \{i1,i2,i3,i4,i5,i7,i8,i9\}$ *and a mapping* m' *of the individuals to places as derived from* m *and* n.

We can show that the firing behavior of our individual approach is compatible with the well-known firing behavior of the collective approach since a firing step in one representation implies a firing step in the respective other one.

Theorem 2 (*Coll* preserves and reflects Firing Behavior)

1. Given a PTI net NI *with transition* $t \in T_{NI}$ *enabled under token selection* $S = (M,m,N,n)$ *with firing step* $NI \xrightarrow{t,S} NI'$*, then* t *is enabled in* $Coll(NI)$ *with firing step* $Coll(NI) \xrightarrow{t} Coll(NI')$.

2. Vice versa, given an enabled transition t *in* $Coll(NI)$ *with* $Coll(NI) \xrightarrow{t} (PN', \mu')$*, there exists a token selection* $S = (M,m,N,n)$ *such that* t *is enabled in* NI *under* S *with firing step* $NI \xrightarrow{t,S} NI'$ *and* $Coll(NI') = (PN', \mu')$.

Proof. Assume $NI = (P,T,pre,post,I,m)$ and $Coll(NI) = (P,T,pre,post,\mu)$.

1. Transition t is enabled in $Coll(NI)$ under μ because $pre(t) \overset{t \text{ enabled}}{=} \sum_{i \in M} m(i) \overset{M \subseteq I}{\leq} \sum_{i \in I} m(i) = \mu$.

Firing changes just the markings, so we have $NI' = (P,T,pre,post,I',m')$ and $Coll(NI') = (P,T,pre,post,\mu')$. We show that μ' is the marking resulting from firing t in $Coll(NI)$.

$$\begin{aligned}
&\mu \ominus pre(t) \oplus post(t) \\
&= \sum_{i \in I} m(i) \ominus \sum_{i \in M} m(i) \oplus \sum_{i \in N} n(i) && \text{(def. } Coll, t \text{ enabled in } NI \text{ under } S\text{)} \\
&= \sum_{i \in I \setminus M} m(i) \oplus \sum_{i \in N} n(i) = \sum_{i \in (I \setminus M) \uplus N} m'(i) && \text{(def. } m' \text{ as in Definition 4)} \\
&= \sum_{i \in I'} m'(i) = \mu' && \text{(defs. } Coll \text{ and } I' \text{ as in Definition 4)}
\end{aligned}$$

2. Because transition t is enabled in $Coll(NI)$ and we have $pre(t) \leq \mu = \sum_{i \in I} m(i)$, we can choose for each $p \in P$ a subset $M_p \subseteq m^{-1}(p)$ such that $|M_p| = pre(t)(p)$. Note that $m(x) = p$ for $x \in M_p$. Similarly, we choose for each $p \in P$ a set N_p such that $|N_p| = post(t)(p)$ and all N_p being mutually disjoint and disjoint to $I \setminus \bigcup_{p \in P} M_p$.

Consider the selection $S = (M,m,N,n)$ with $M = \bigcup_{p \in P} M_p$, $N = \bigcup_{p \in P} N_p$, and function $n\colon N \to P$ with $n(x) = p$ for $x \in N_p$. The transition t is enabled in NI under S because $\sum_{i \in M} m(i) = \sum_{p \in P} \sum_{i \in M_p} m(i) = \sum_{p \in P} |M_p| \cdot p = \sum_{p \in P} pre(t)(p) \cdot p = pre(t)$ and analogously for N, n, and

$post$. For the firing step $NI \xrightarrow{t,S} NI'$, we have $NI' = (P,T,pre,post,I',m')$ according to Definition 4 and $PN' = (P,T,pre,post)$ because firing changes the marking, only. We show for $Coll(NI') = (PN',\hat{\mu})$ that $\mu' = \hat{\mu}$. The arguments are analogous to the ones for the equations for item 1. $\mu' = \mu \ominus pre(t) \oplus post(t) = \sum_{i\in I} m(i) \ominus \sum_{i\in M} m(i) \oplus \sum_{i\in N} n(i) = \sum_{i\in I\setminus M} m(i) \oplus \sum_{i\in N} n(i) = \sum_{i\in (I\setminus M)\uplus N} m'(i) = \sum_{i\in I'} m'(i) = \hat{\mu}$. □

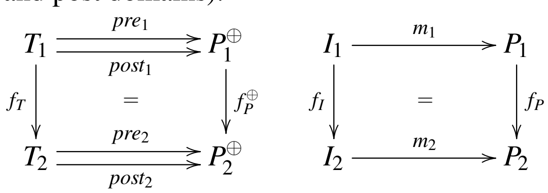
Corollary 1 (Equivalent Firing Behavior) *Given PTI nets $NI_1 \approx NI_2$ and a firing step $NI_1 \xrightarrow{t,S} NI_1'$. Then there is a corresponding firing step $NI_2 \xrightarrow{t,S'} NI_2'$ with $NI_1' \approx NI_2'$.*

Proof. By Lemma 1 we have $Coll(NI_1) = (PN,\mu) = Coll(NI_2)$ and by Theorem 2 there is a firing step $(PN,\mu) \xrightarrow{t} (PN,\mu') = Coll(NI_1')$, implying a reflected step $NI_2 \xrightarrow{t,S'} NI_2'$ with $Coll(NI_2') = (PN,\mu') = Coll(NI_1')$. Hence, by Lemma 1 there is $NI_1' \approx NI_2'$. □

2.3 Transformation of P/T Nets with Individual Tokens

The structure of a P/T system can be changed by the application of transformation rules using the double pushout (DPO) approach (see [EEPT06]). For the definition of transformation rules for PTI nets we need the following definition of PTI net morphisms.

Definition 5 (PTI Net Morphisms and Category **PTINets**) Given two PTI nets $NI_i = (P_i,T_i,pre_i,post_i,I_i,m_i)$, $i \in \{1,2\}$, a PTI net morphism is a triple of functions $f = (f_P : P_1 \to P_2, f_T : T_1 \to T_2, f_I : I_1 \to I_2) : NI_1 \to NI_2$, such that the following diagrams commute (componentwise for pre and post domains):



or, explicitly, that $f_P^{\oplus} \circ pre_1 = pre_2 \circ f_T$, $f_P^{\oplus} \circ post_1 = post_2 \circ f_T$, and $f_P \circ m_1 = m_2 \circ f_I$.
The category **PTINets** consists of all PTI nets as objects with all PTI net morphisms.

Remark 2 (Choice of PTI morphisms) *We are aware that there exist several different reasonable definitions of morphisms for P/T nets in the algebraic representation with monoids of [MM90]. Although the P/T morphisms from [EEPT06], on which our PTI morphisms are based, are restricted in contrast to more general definitions of P/T morphisms, e. g. consisting of arbitrary monoid homomorphisms for the component on place monoids and partial functions for the transition components, we chose the current definition. For our simple PTI morphisms consisting of total functions, pushouts simply can be constructed componentwise, leading to $\mathcal{M}$-adhesive categories with a class $\mathcal{M}$ of injective morphisms. This is no longer valid for more general morphisms as mentioned above.*

In [MGE+10], we show that PTI morphisms with injective token component preserve firing steps. For a token-injective morphism $f : NI_1 \to NI_2$, and a firing step (t,S) in NI_1 is is not possible to canonically construct a selection S' such that $(f_T(t),S')$ is a firing step in NI_2, but we show that such a step exists. The reason of this is that some newly created token in N_S

may also exist in $I_2 \setminus f_T(M_S)$ so that the subset of conflicting tokens in N_S has to be replaced isomorphically such that N_S becomes disjoint to $I_2 \setminus f_T(M_S)$. Up to a suitable renaming of these conflicting individual tokens, token-injective PTI morphisms are simulations of firing behavior.

Definition 6 (PTI Transformation Rules) A PTI transformation rule is a span of injective **PTINets** morphisms $\rho = (L \overset{l}{\leftarrow} K \overset{r}{\rightarrow} R)$.

Definition 7 (PTI Transformation) Given a PTI transformation rule $\rho = (L \overset{l}{\leftarrow} K \overset{r}{\rightarrow} R)$ and a PTI net NI_1 with a PTI net morphism $f : L \rightarrow NI_1$, called the match, a direct PTI net transformation $NI_1 \overset{\rho,f}{\Longrightarrow} NI_2$ from NI_1 to the PTI net NI_2 is given by the following double-pushout (DPO) diagram in the category **PTINets**:

$$\begin{array}{ccccc} L & \xleftarrow{\quad l \quad} & K & \xrightarrow{\quad r \quad} & R \\ {\scriptstyle f}\downarrow & (\mathbf{PO_1}) & \downarrow & (\mathbf{PO_2}) & \downarrow{\scriptstyle f^*} \\ NI_1 & \longleftarrow & NI_0 & \longrightarrow & NI_2 \end{array}$$

The application of a rule with a given match following the DPO approach means that we compute first a pushout complement to obtain pushout (PO_1) and then the pushout (PO_2) in **PTINets**. Note that pushouts and therefore the result of a rule application are unique only up to isomorphism. Intuitively, everything that is matched from the left-hand side L that does not have a preimage in the interface K is deleted leading to a context net NI_0. Then the right-hand side R is glued to the context NI_0 along the interface K leading to the result NI_2 of the transformation.

Remark 3 (Construction of Pushouts and Pushout Complements) *Pushouts in the category* **PTINets** *can be constructed componentwise in* **PTNets** *and* **Sets***, where the marking function of the pushout PTI net is induced by the pushout of the token sets. Since* $(\mathbf{PTNets}, \mathcal{M}_1)$ *and* $(\mathbf{Sets}, \mathcal{M}_2)$ *with classes* $\mathcal{M}_1$ *of injective P/T net morphisms and* $\mathcal{M}_2$ *of injective functions are* $\mathcal{M}$*-adhesive categories (see [EEPT06]) they have unique pushout complements along* $\mathcal{M}$*-morphisms. Thus, also* **PTINets** *has unique pushout complements along injective morphisms.*

Example 3 (Transformation of PTI Nets) *Figure 2 shows the application of a PTI rule newClient which connects a new client with a data capacity of three tokens to an existing switch. For simplicity reasons all morphisms in the DPO diagram are inclusions. The left-hand side L of the rule is matched to a PTI net* $Network_1$ *which already contains one Client. Since the rule is non-deleting we obtain a context net* $Network_0$ *which equals the original network. The result of the transformation is a new PTI net* $Network_2$ *where the new client has been connected to the switch.*

3 Main Results for P/T Nets with Individual Tokens

In this section, we present main results for transformation systems of PTI nets following from the properties and results of weak adhesive high-level replacement (HLR) systems [EEPT06]. The latter are based on the notion of adhesive categories introduced in [LS04]. The results for

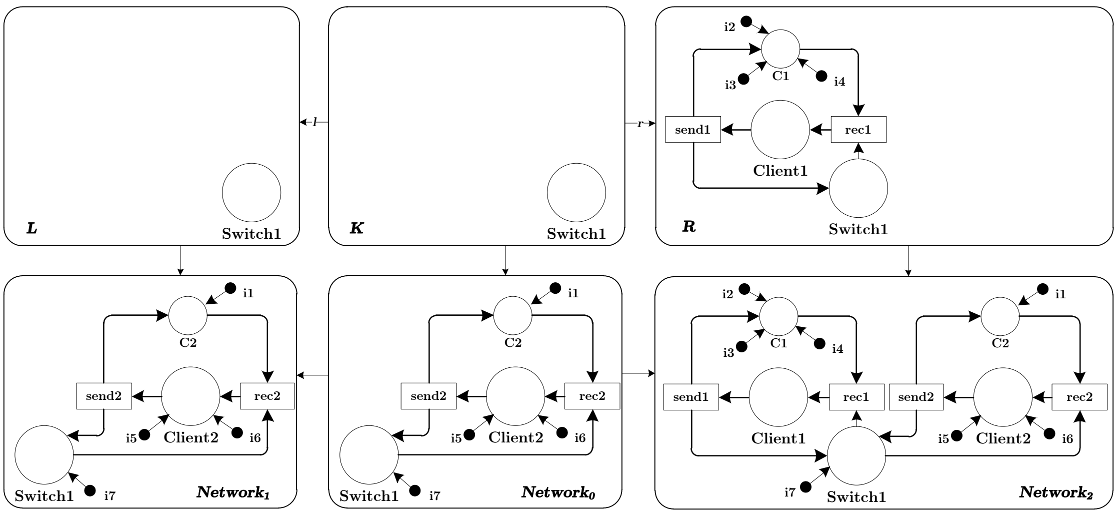


Figure 2: PTI transformation $Network_1 \stackrel{newClient}{\Longrightarrow} Network_2$

weak adhesive HLR systems are also valid for $\mathscr{M}$-adhesive transformation systems [EGH10], which are a generalization that has been triggered by [Hei10].

3.1 P/T Nets with Individual Tokens as $\mathscr{M}$-adhesive Category

In this paper, we use the notion of $\mathscr{M}$-adhesive category [EGH10] which is short for vertical weak adhesive HLR category. In $\mathscr{M}$-adhesive categories Van Kampen (VK) squares only need to satisfy the vertical VK-property, i. e. the VK-property has to hold for cubes where the vertical morphisms are in $\mathscr{M}$. In contrast, for a weak adhesive HLR categories it is required that the VK-property does also hold for cubes, where the horizontal morphisms are in $\mathscr{M}$. However, as shown in [EGH10] all the main results of [EEPT06] are still valid for $\mathscr{M}$-adhesive categories.

Theorem 3 (PTINets is $\mathscr{M}$-adhesive) *The category* $(\mathbf{PTINets}, \mathscr{M}_{inj})$ *is an* $\mathscr{M}$*-adhesive category where* $\mathscr{M}_{inj} = \{f \in Mor_{\mathbf{PTINets}} \mid f_P, f_T, f_I \text{ injective}\}$.

Proof (Idea). We already know from [EEPT06] that the category of P/T nets (without markings) $(\mathbf{PTNets}, \mathscr{M}')$ is weak adhesive HLR and hence also $\mathscr{M}$-adhesive with $\mathscr{M}'$ being the class of all injective Petri net morphisms. We construct a comma category over $(\mathbf{PTNets}, \mathscr{M}')$ and an individual marking functor such that this comma category is isomorphic to the $\mathscr{M}$-category of PTI nets with possibly infinite tokens sets and the class $\mathscr{M}_{inj}$ of injective PTI net morphisms. From a construction theorem in [Pra08] follows that this comma category is $\mathscr{M}$-adhesive. A more detailed proof can be found in [MGE$^+$10].

Finally, the full subcategory **PTINets** of PTI nets with finite token sets is $\mathscr{M}$-adhesive as well for the class $\mathscr{M}_{inj}$. This is guaranteed by another construction theorem from [Pra08], as the inclusion functor from $\mathbf{Sets}_{\text{fin}}$ to **Sets** preserves pushouts and pullbacks. □

Using this theorem, we can apply the important results for analyzing transformations from

[EEPT06] to transformations of PTI nets, e. g. the theorems about independent rule applications (Local Church-Rosser), concurrent rule applications, and local confluence of transformation systems.

$\mathcal{M}$-adhesive transformation systems guarantee unique results of rule applications (up to isomorphisms). Note that morphisms of **PTSys** rules have to be marking-strict in order to obtain an $\mathcal{M}$-adhesive **PTSys** transformation system [PEHP08]. This requirement is not necessary for an $\mathcal{M}$-adhesive **PTINets** transformation system, allowing us to simulate the firing behavior of PTI nets with direct transformations as we show in Subsection 3.3.

3.2 Gluing Condition for P/T Nets with Individual Tokens

In order to be able to decide whether a rule is applicable at a certain match, we formulate a gluing condition for PTI nets, such that there exists a pushout complement of the left rule morphism and the match if (and only if) they fulfill the gluing condition.

Definition 8 (Gluing Condition in **PTINets**) Given a PTI rule $\rho = (L \overset{l}{\leftarrow} K \overset{r}{\rightarrow} R)$, a PTI net NI and a PTI morphism $f : L \rightarrow NI$ (see the left part of Figure 3), we define the set of identification points (i. e. all elements in L that are mapped non-injectively by f) $IP = IP_P \cup IP_T \cup IP_I$ with

- $IP_P = \{x \in P_L \mid \exists x' \neq x \colon f_P(x) = f_P(x')\}$,
- $IP_T = \{x \in T_L \mid \exists x' \neq x \colon f_T(x) = f_T(x')\}$,
- $IP_I = \{x \in I_L \mid \exists x' \neq x \colon f_I(x) = f_I(x')\}$,

the set of dangling points (i. e. all places in L that would leave a dangling arc, if deleted) $DP = DP_T \cup DP_I$ with

- $DP_T = \{p \in P_L \mid \exists t \in T_{NI} \setminus f_T(T_L) \colon f_P(p) \in ENV(t)\}$,
- $DP_I = \{p \in P_L \mid \exists i \in I_{NI} \setminus f_I(I_L) \colon f_P(p) = m_{NI}(i)\}$,

and the set of gluing points (i. e. all elements in L that have a preimage in K) $GP = l_P(P_K) \cup l_T(T_K) \cup l_I(I_K)$.

We say that ρ and f satisfy the gluing condition if $IP \cup DP \subseteq GP$

For the following theorem, we consider the $\mathcal{M}$-adhesive category $(\mathbf{PTINets}, \mathcal{M})$ whose morphism class $\mathcal{M}$ contains all injective morphisms.

Theorem 4 (Gluing Condition for PTI Transformation) *Given a PTI rule $\rho = (L \overset{l}{\leftarrow} K \overset{r}{\rightarrow} R)$ with $l, r \in \mathcal{M}$ and a match $f \colon L \rightarrow NI$ into a PTI net $NI = (N, I, m \colon I \rightarrow P_{NI})$. The rule ρ is applicable on match f, i. e. there exists a (unique up to isomorphisms) pushout complement NI_0 in the diagram in Figure 3, iff ρ and f satisfy the gluing condition in* **PTINets***.*

Proof (Idea). In [MGE+10], we show that the gluing condition from Definition 8 is equivalent to the categorical gluing condition from [EEPT06] for $\mathcal{M}$-adhesive transformation systems, which states that the boundary of an initial pushout construction over the match has to be preserved by the rule. □

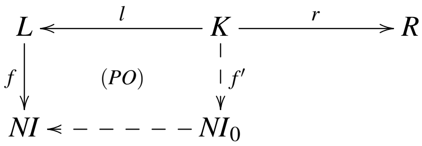


Figure 3: Diagram of a mached rule and the possible pushout complement

Example 4 (Gluing Condition) *Consider the transformation rule deleteClient for PTINets depicted in Figure 4 and an inclusion match f into the PTI net $(SimpleNetwork, I, m)$ shown in Figure 1. The rule deleteClient and match f do not satisfy the gluing condition because due to the fact that the individuals i2, i3 and i4 are not matched by the rule, the place Client1 is a dangling point and therefore it should have a preimage in the interface K of the rule (i. e. it should be a gluing point) in order to satisfy the gluing condition. Since this is not the case the rule is not applicable with the given match.*

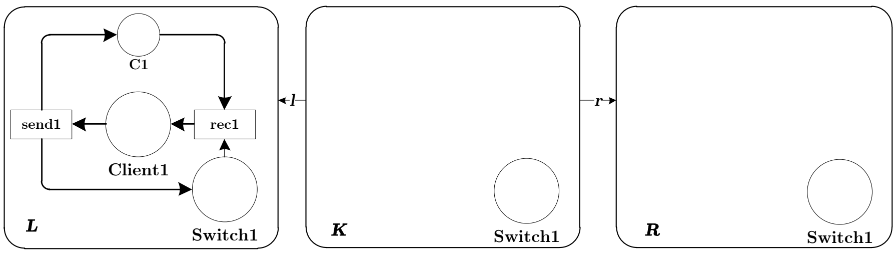


Figure 4: Transformation rule *deleteClient* for PTI nets

In contrast, the PTI transformation rule newClient shown in Figure 2 together with the match described in Example 3 satisfies the gluing condition, because since the match is injective there are no identification points, and the only dangling point Switch1 is a gluing point. Therefore the rule newClient can be applied with the given match.

3.3 Correspondence of Transition Firing and Rule Applications

An interesting aspect of the possibility to formulate marking-changing rules in **PTINets** is that rules can simulate firing steps of transitions. We give a construction for transition rules that simulate a firing step of some transition under a specific token selection and show that firing of a transition corresponds to an application of a transition rule and vice versa.

Definition 9 (PTI Transition Rules) We define the *transition rule* for a transition $t \in T$ of a PTI net $NI = (P, T, pre, post, I, m)$, enabled under a token selection $S = (M, m, N, n)$, as the rule $\rho(t, S) = (L_t \overset{l}{\leftarrow} K_t \overset{r}{\rightarrow} R_t)$ with

- the common fixed net structure $PN_t = (P_t, \{t\}, pre_t, post_t)$, where $P_t = ENV(t), pre_t(t) = pre(t)$ and $post_t(t) = post(t)$,
- $L_t = (PN_t, M, m_t : M \to P_t)$, with $m_t(x) = m(x)$,

- $K_t = (PN_t, \emptyset, id_\emptyset)$,
- $R_t = (PN_t, N, n_t : N \to P_t)$, with $n_t(x) = n(x)$,
- l, r being the obvious inclusions on the rule nets.

Example 5 (Simulation of Firing Behavior by Rule-Based Transformation) *Consider again the firing of the transition send2 in Example 2. The firing step can be simulated by application of the transition rule $\rho(send2, S)$ shown in Figure 5 to the PTI net $(SimpleNetwork, I, m)$ in Figure 1 leading to the same result.*

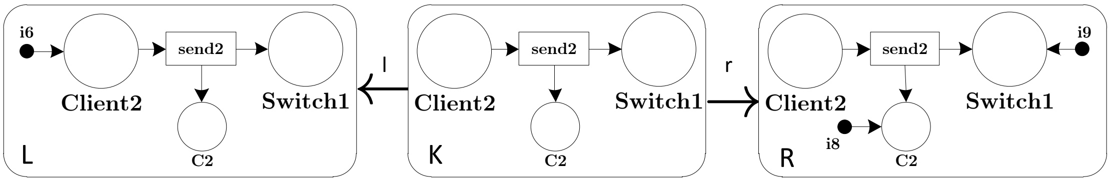


Figure 5: Transition rule $\rho(send2, S)$

Theorem 5 (Correspondence between Firing Steps and Direct DPO Transformations of PTI Nets)

1. Each firing step $NI \overset{t,S}{\rightarrowtail} NI'$ via token selection $S = (M, m, N, n)$ corresponds to an induced direct transformation $NI \overset{\rho(t,S),f}{\Longrightarrow} NI'$ via the transition rule $\rho(t,S)$, where the match $f : L_{\rho(t,S)} \to NI$ is an inclusion.

2. Each direct transformation $NI \overset{\rho(t,S),f}{\Longrightarrow} NI_1$ via some transition rule $\rho(t,S)$ with $t \in T_{NI}$, token selection $S = (M, m, N, n)$, and injective match $f : L_{\rho(t,S)} \to NI$, implies that the transition $f_T(t)$ is enabled in NI under some token selection $\bar{S}$ with firing step $NI \overset{f_T(t),\bar{S}}{\rightarrowtail} NI^*$ such that $NI^* \cong NI_1$.

Proof. In the following let $NI = (PN, I, m)$, $NI' = (PN', I', m')$, and $NI_i = (PN_i, I_i, m_i)$.
Part 1. Consider the DPO diagram in Figure 6 with inclusions d and d', i. e. $PN = PN_0 = PN_1$.

$$\begin{array}{ccccc} L = (PN_t, M, m_t) & \xleftarrow{l} & K = (PN_t, \emptyset, \emptyset) & \xrightarrow{r} & R = (PN_t, N, n_t) \\ \downarrow f & (\mathbf{PO}) & \downarrow & (\mathbf{PO}) & \downarrow f^* \\ NI = (PN, I, m) & \xleftarrow{d} & NI_0 = (PN_0, I_0, m_0) & \xrightarrow{d'} & NI_1 = (PN_1, I_1, m_1) \end{array}$$

Figure 6: DPO transformation diagram in **PTINets** for $\rho(t,S)$ applied to NI

This diagram exists by Theorem 4 because there are no identification points (f is injective) and all dangling points are gluing points ($l_P = id_{P_t}$, i. e. no places are deleted). Because pushouts in **PTINets** can be constructed componentwise for the net and the token components, we have $I_0 = I \setminus M$ and $I_1 = I_0 \uplus (N \setminus \emptyset)$ as in the DPO diagram of the **Sets** components in Figure 7. By assumption t is enabled under S, so we have that $(I \setminus M) \cap N = \emptyset$ and therefore $I_1 = (I \setminus M) \cup N$.

For m_1 as induced morphism for the pushout object I_1 follows that

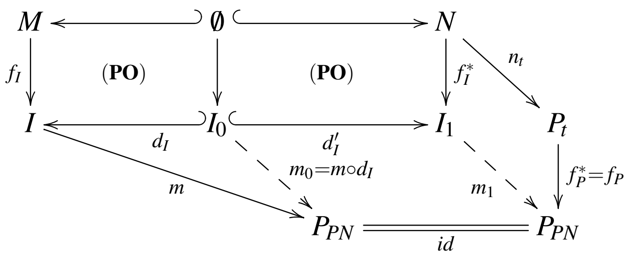
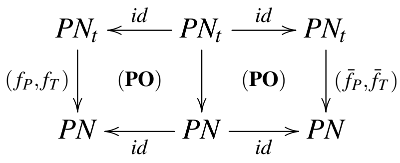


Figure 7: DPO diagram in **Sets** for the token components in Figure 6

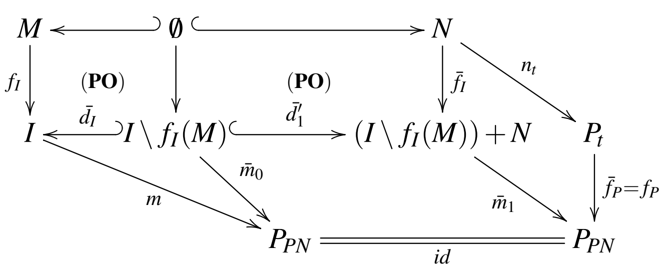
$$m_1(x) = \begin{cases} m_0(x) = m(x) \text{ for } x \in I \setminus M \\ n_t(x) = n(x) \text{ for } x \in N \end{cases}$$

hence $I_1 = I', m_1 = m'$ according to Definition 4 and therefore $NI_1 = NI'$. This proves the existence of the direct transformation $NI \xRightarrow{\rho(t,S),f} NI'$.

Part 2. Given a direct transformation $NI \xRightarrow{\rho(t,S),f} NI_1$ as in the DPO diagrams in Figure 6 and Figure 7, there is also a direct transformation $NI \xRightarrow{\rho(t,S),f} \overline{NI}$ with $\overline{NI} = (PN, (I \setminus f_I(M)) + N)$ given by the componentwise DPOs in Figure 8a by standard category theory and Figure 8b by construction of pushout complements and pushouts in **Sets** (see [EEPT06]) where we choose the injection $\bar{d}'_I$ to be an inclusion. Then there is $\overline{NI} \cong NI_1$ by uniqueness of pushouts and pushout complements in **PTINets**.



(a) DPO in **PTNets**



(b) DPO in **Sets**

Figure 8: Componentwise DPO diagrams in **PTNets** and **Sets**

Then $f_T(t) \in T_{NI}$ is enabled under a token selection $\bar{S} = (\bar{M}, \bar{m}, \bar{N}, \bar{n})$ with $\bar{M} = f_I(M)$, $\bar{m} = m$, $\bar{N} = \bar{f}_I(N)$ and $\bar{n} = \bar{m}_1|_{\bar{N}}$ if

1. $\bar{M} \subseteq I$,
2. $\bar{n} : \bar{N} \to P_{PN}$,
3. $(I \setminus \bar{M}) \cap \bar{N} = \emptyset$, and
4. $\sum_{i \in \bar{M}} \bar{m}(i) = pre_{NI}(f_T(t))$
5. $\sum_{i \in \bar{N}} \bar{n}(i) = post_{NI}(f_T(t))$

Items 1 and 2 hold by construction via image and restriction. Item 3 follows from the fact that the coproduct $(I \setminus \bar{M}) + N$ is a disjoint union in **Sets** and $\bar{N} = \bar{f}_I(N)$ is exactly the part of that set which is not in $I \setminus \bar{M}$. It remains to show items 4 and 5:

$$\begin{aligned}
&\sum_{i\in\bar{M}} \bar{m}(i) = \sum_{i\in f_I(M)} \bar{m}(i) \\
&= \sum_{i\in M} m \circ f_I(i) && (f_I \text{ inj.}, \bar{m} = m) \\
&= \sum_{i\in M} f_P \circ m_t(i) && (f \ \textbf{PTINets}\text{-morphism}) \\
&= f_P^{\oplus} \sum_{i\in M} m_t(i) = f_P^{\oplus} \sum_{i\in M} m(i) && (\forall i \in M : m_t(i) = m(i) \text{ by def. of } \rho(t,S)) \\
&= f_P^{\oplus} \circ pre_{PN_t}(t) && (t \text{ enabled under } S \text{ in } L_{\rho(t,S)}) \\
&= pre_{NI} \circ f_T(t) && (f \ \textbf{PTINets}\text{-morphism})
\end{aligned}$$

and analogously,

$$\begin{aligned}
&\sum_{i\in\bar{N}} \bar{n}(i) = \sum_{i\in \bar{f}_I(N)} \bar{n}(i) \\
&= \sum_{i\in N} \bar{m}_1 \circ \bar{f}_I(i) && (\bar{f}_I \text{ inj.}, \bar{n} = \bar{m}_1|_{\bar{f}_I(N)}) \\
&= \sum_{i\in N} f_P \circ n_t(i) && (\bar{f} = (\bar{f}_P, \bar{f}_T, \bar{f}_I) \ \textbf{PTINets}\text{-morphism}, \bar{f}_P = f_P) \\
&= f_P^{\oplus} \sum_{i\in N} n_t(i) = f_P^{\oplus} \sum_{i\in N} n(i) && (\forall i \in N : n_t(i) = n(i) \text{ by def. of } \rho(t,S)) \\
&= f_P^{\oplus} \circ post_{PN_t}(t) && (t \text{ enabled under } S \text{ in } L_{\rho(t,S)}) \\
&= post_{NI} \circ f_T(t) && (f \ \textbf{PTINets}\text{-morphism})
\end{aligned}$$

So we have that $f_T(t)$ is enabled under $\bar{S}$ and we obtain a firing step $NI \xrightarrow{f_T(t),\bar{S}} NI^*$ where NI^* has the same net part PN and the follower marking (I^*, m^*) with $I^* = (I \setminus \bar{M}) \cup \bar{N}$ and

$$m^*(x) = \begin{cases} \bar{m}(x) = m(x) = \bar{m}_1(x)|_{I\setminus\bar{M}} & \text{, if } x \in I \setminus \bar{M}; \\ \bar{n}(x) = \bar{m}_1(x)|_{\bar{N}} & \text{, if } x \in \bar{N}. \end{cases}$$

Now, by the fact that $\bar{d}_I'$ is an inclusion we have

$$I^* = (I \setminus \bar{M}) \cup \bar{N} = (I \setminus f_I(M)) \cup \bar{f}_I(N) = (I \setminus f_I(M)) + N$$

and the marking function $m^* : I^* \to P_{PN}$ maps the individuals exactly like $\bar{m}_1 : (I \setminus f_I(M)) + N \to P_{PN}$. So we have $NI^* = \overline{NI}$ and hence $NI^* \cong NI_1$. □

The encoding of PTI transition rules and the correspondence between the firing of PTI nets and the application of transition rules stated in the theorem above are very close to those presented in [Kre81]. A difference, however, of our encoding is the fact that the transition rules are encoded directly as PTI transformation rules rather than as graph transformation rules like in [Kre81].

This allows us to use the transition rules for analysis in PTI transformation systems as presented in the next subsection.

A generalization of Petri nets to graph grammars is presented in [CM95] such that transitions correspond to rules and firing steps to rule applications. This approach uses an encoding of transitions as graph rules similar to Definition 9 and the transition productions of [Kre81] with the difference that they contain only the individual tokens as typed graph nodes where the types represent the places marked by the tokens. The authors of [CM95] mention a subtle mismatch of the encoding as a conceptual problem, i. e. that the indistinguishable tokens of the multiset marking in the actual net are more abstract than the tokens with distinguishable individuals in the graph representation. Because of several possible matches for the individuals, there are different transformations representing the same unique firing step of a transition and there are many grammars representing the same net.

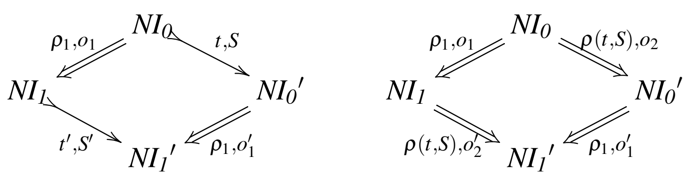
Although both constructions have inspired the transition rules in this paper, it is not our ambition to formalize a strict simulation relation between a PTI net's behavior and an – in some sense – equivalent (net) grammar. We rather use the correspondence result of Theorem 5 to relate arbitrary net transformation steps with firing steps as we show in the next section.

3.4 Independence of Token-Firing and Rule Application

For P/T systems, [EHP+07] defines parallel and sequential independence of a transformation step and a firing step and proves results that are similar to the Local Church-Rosser Theorem of [EEPT06], which relates sequential and parallel independence of rule applications.

As we have shown in Theorem 5 we are able to express the firing of PTI nets by application of transition rules. This allows us to immediately use the results for $\mathcal{M}$-adhesive transformation systems [EEPT06, EGH10] for the analysis of the independence of firing steps and rule applications. We obtain a notion of parallel independence of a rule application and a firing step for PTI nets by relying on the definition of these properties for the corresponding transition rule.

Definition 10 (Parallel Independence of Rule Applications and Firing Steps) A transformation step $NI_0 \overset{\rho_1,o_1}{\Longrightarrow} NI_1$ and a firing step $NI_0 \overset{t,S}{\rightarrowtail} NI_0'$ (see the top of Figure 9a) are parallel independent iff the rule applications (ρ_1, o_1) and $(\rho(t,S), o_2)$ are parallel independent (see Figure 9b), where $(\rho(t,S), o_2)$ is defined according to item 1 of Theorem 5.



(a) Independence diagram for a rule application and a firing step

(b) Corresponding local Church-Rosser diagram

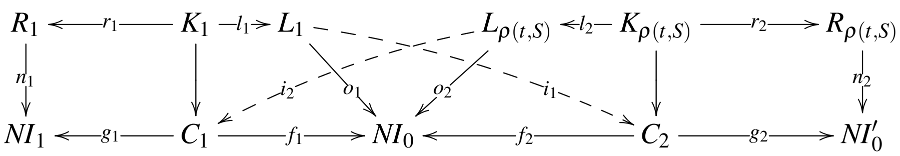
Figure 9: Independence of Rule Applications and Firing Steps

Remark 4 (Sequential Independence of Rule Applications and Firing Steps) *Analogously to the parallel independence it is possible to define the sequential independence of rule applications and firing steps by defining that $NI_0 \overset{\rho_1}{\Longrightarrow} NI_1 \overset{t,S}{\rightarrowtail} NI_2$ respectively $NI_0 \overset{t,S}{\rightarrowtail} NI_1 \overset{\rho_1}{\Longrightarrow} NI_2$ are sequentially independent iff $NI_0 \overset{\rho_1}{\Longrightarrow} NI_1 \overset{\rho(t,S)}{\Longrightarrow} NI_2$ respectively $NI_0 \overset{\rho(t,S)}{\Longrightarrow} NI_1 \overset{\rho_1}{\Longrightarrow} NI_2$ are sequentially independent.*

Now, we can transfer the relations that the Local Church-Rosser Theorem states between parallel and sequentially independent rule applications to parallel and sequentially independent rule applications and firing steps. With this result, we have a criterion to decide for rule applications and firing steps whether they can occur independently in any order with the same result.

Theorem 6 (Local Church Rosser for Rule Applications and Firing Steps) *Given a direct transformation $NI_0 \overset{\rho_1,o_1}{\Longrightarrow} NI_1$ and a firing step $NI_0 \overset{t,S}{\rightarrowtail} NI'_0$ that are parallel independent (see the top of Figure 9a), then there exists a transition $t' \in T_1$ enabled under some token selection S' leading to $NI_1 \overset{t',S'}{\rightarrowtail} NI'_1$ and a direct transformation $NI'_0 \overset{\rho_1,o'_1}{\Longrightarrow} NI'_1$.*

Proof. Parallel independence of (ρ_1, o_1) and (t,S) by Definition 10 means that (ρ_1, o_1) and $(\rho(t,S), o_2)$ are parallel independent rule applications. This implies by Theorem 5.12 in [EEPT06] that there are sequentially independent transformations $NI_0 \overset{\rho_1,o_1}{\Longrightarrow} NI_1 \overset{\rho(t,S),o'_2}{\Longrightarrow} NI'_1$ and $NI_0 \overset{\rho(t,S),o_2}{\Longrightarrow} NI'_0 \overset{\rho_1,o'_1}{\Longrightarrow} NI'_1$ as depicted in Figure 9b. The match o'_2 is given by the composition $o'_2 = g_1 \circ i_2$ where i_2 with $o_2 = f_1 \circ i_2$ is given by the parallel independence of the rules (cf. Theorem 5.12 in [EEPT06]).



Now, injectivity of o_2 implies injectivity of i_2, and injectivity of r_1 implies injectivity of g_1 because $\mathcal{M}$-morphisms are closed under pushouts. So we have that $o'_2 = g_1 \circ i_2$ is injective and thus by item 2 of Theorem 5 we have that $t' = o'_{2,T}(t)$ is enabled under some token selection S' with firing step $NI_1 \overset{t',S'}{\rightarrowtail} NI^*_1$ such that $NI^*_1 \cong NI'_1$. □

Remark 5 *Due to the fact that a rule sequence $NI_1 \overset{\rho_1}{\Longrightarrow} NI_2 \overset{\rho_2}{\Longrightarrow} NI_3$ is sequentially independent iff $NI_1 \overset{\rho_1^{-1}}{\Longleftarrow} NI_2 \overset{\rho_2}{\Longrightarrow} NI_3$ are parallel independent, the theorem above can be easily extended to cover also the analogous statement for sequentially independent firing and rule application. This argumentation has similarly been used in [EHP⁺07].*

Example 6 (Independence of Rule Application and Firing Step) *Consider the PTINet $Network_0$ in the top-left corner of Figure 10. We can add a new client Client1 by applying the rule newClient in the top of Figure 2 with an inclusion match morphism o. Moreover, we can fire the transition send2 under a selection S as described in Example 2. The rule application and the firing step are parallel independent which means that the diagram can be completed with sequentially independt dent sequences of rule application and firing steps as shown in Figure 10.*

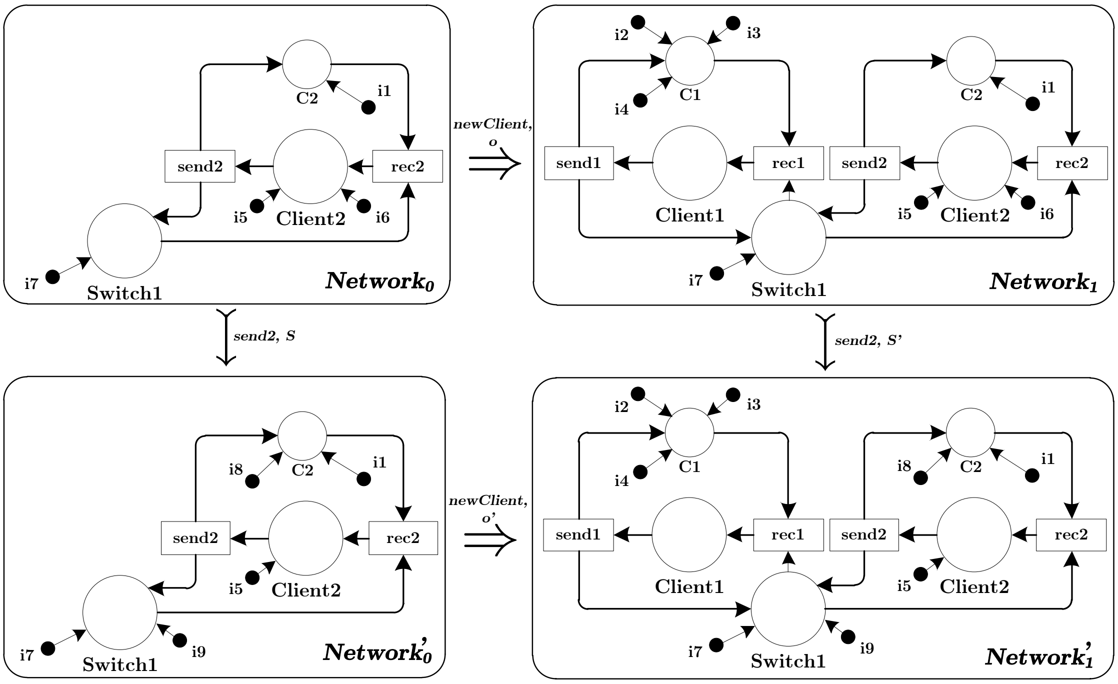


Figure 10: Independent application of *newClient* and firing of *send2*

4 Conclusion and Future Work

In this paper, we have presented place/transition nets with individual tokens (PTI nets) together with a rule-based transformation by instantiation of $\mathcal{M}$-adhesive transformation systems. The individual token approach of PTI nets overcomes some technical restrictions of reconfigurable P/T systems and provides an appropriate representation of marking-changing rules.

As a main result, we have shown that the category of PTI nets together with the class of all injective morphisms form an $\mathcal{M}$-adhesive category, where the framework of $\mathcal{M}$-adhesive categories is a slight generalization of weak adhesive high-level replacement (HLR) categories. This allows us to use the analysis results for weak adhesive HLR systems from [EEPT06] for PTI transformation systems, and we obtain a necessary and sufficient gluing condition for the application of PTI transformation rules. Moreover, we have shown that firing steps in PTI nets are equivalent to applications of special transformation rules, called transition rules, simulating a firing step by changing the marking of the places in the environment of the fired transition accordingly. With this correspondence of firing steps and rule applications, we are able to define the notions of parallel and sequential independence of a PTI firing step and a rule application by using the definitions of independence for rule applications from [EEPT06]. This allows to show a local Church-Rosser result for rule application and token firing based on the corresponding results in $\mathcal{M}$-adhesive categories and is the basis for further conflict analysis based on critical pairs.

In our technical report [MGE$^+$10], we extend our approach of Petri nets with individual tokens to transformation systems of algebraic high-level nets with individual tokens (AHLI nets) based on rule-based transformations of algebraic high-level nets from [PER95]. For AHLI nets we obtain similar results as for PTI nets. That is, $(\mathbf{AHLINets}, \mathcal{M})$ with the class of all injective morphisms with isomorphic data part is an $\mathcal{M}$-adhesive category. Moreover, we have a sufficient and necessary gluing condition for the applicability of AHLI rules, and it is also possible to express the firing of AHLI nets by application of AHLI transition rules.

We employ AHLI nets in our modeling case study for Skype in [HM10] for realizing multicasting to transmit specific data between groups of Skype clients by marking-changing rules according to [BEE+09]. In that case study, we use the algebraic data type part in order to represent the clients' identities and the communicated data.

Due to the categorical characterization of independence in this paper in contrast to [EHP+07], the results only rely on the correspondence as stated in Theorem 5. Therefore, the Local Church-Rosser Theorem for AHLI rule applications and firing steps can be shown completely analogously because of the correspondence between the firing of AHLI nets and the application of AHLI transition rules [MGE+10]. Moreover, it is possible to transfer similar results for transformations like the theorems for concurrency and local confluence based on critical pairs [EEPT06] to transformations mixed with firing steps of P/T and AHL nets with individual tokens.

In [EHL10], the results of [EEPT06] concerning parallel and concurrent rules have been lifted to transformation systems with nested application conditions (see also [HP09]). The additional property for $\mathcal{M}$-adhesive categories that is needed for these results is a suitable $\mathcal{E}$-$\mathcal{M}$ factorization (and binary coproducts which we already have by cocompleteness). One possibility to achieve this requirement is the restriction to finite PTI and AHLI nets, because as shown in [BEGG10], the restriction of an $\mathcal{M}$-adhesive category to all its finite objects has extremal $\mathcal{E}$-$\mathcal{M}$ factorizations.

Another powerful concept is the amalgamation of rules (with application conditions) over a bundle of matches [GEH10] which can be used to realize multicasting of data tokens in high-level nets [BEE+09]. In order to instantiate the results in that article to our Petri net categories we need to show that they have so-called effective pushouts.

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